Quantitative Management

Master Analyse et Calcul Economiques - Université Paris-Dauphine Ingénierie Economique et Financière

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2008
What is QM?

– Most of asset manager: rule of the thumb, experience and luck...
– Now, lots of models applied to decide on winning trades: quantitative managers.

Globally, asset management = methods to handle funds and portfolio based on various assets:

1. Equity assets:
   – technical analysis, now used for market timing. Originally:
     "Technical analysis is the study of market action, primarily through the use of charts, for the purpose of forecasting future price trends."
   – Factor based models: classical Capital Asset Pricing Model and $\alpha$ trading; more complicated versions and Roll’s Arbitrage Pricing Theory using a multifactor approach...
– Portfolio insurance: e.g. Stop-Loss, Option Based Portfolio Insurance (OBPI) and Constant Portfolio Protection Insurance (CPPI).
– Black & Litterman’s approach to Markowitz problem.
– ...

2. Bond Markets and FX:
– "No arbitrage" Taylor rules: linking monetary policy and the yield curve.
– Principal component analysis for market scenario building. PCA factors. Bond pricing with a limited number of factors.
– Cointegration: trading on mean reverting spreads.
– Macroeconomic prediction in a data-rich environment: trading on economics.
– ...

Each of these approaches try to deal with the Markowitz’s enormous lack: how do I deal with the expected return?
Purposes of this class

Not to review each of these elements (most of them may have been presented during other courses), but to focus on several:

1. Portfolio insurance: SL, OBPI and CPPI.
2. Review the PCA methodology and its various applications.
3. Use basic cointegration results to trade on mean reverting spreads.
**Why care about strategic allocation?**

Now, a few words about Markowitz. Basically, the problem is:

\[
\text{max } \mathbb{E}_t[U(w^\top r_{t+1})].
\] (1)

Whenever:

1. returns are gaussian

\[
r_{t+1}, t \sim N(\mu, \Sigma)
\] (2)

2. utility is exponential

\[
U(x) = -\exp\{-ax\}
\] (3)

This problem is easy to solve, because:

\[
\mathbb{E}_t[e^{-aw^\top r_{t+1}}] = -e^{-aw^\top \mu + a^2 \frac{1}{2} w^\top \Sigma w}
\] (4)
which is equivalent to solving:

$$\max w^\top \mu - \frac{a}{2} w^\top \Sigma w$$  \hspace{1cm} (5)$$

Usual solution to Markowitz problem and to mean-variance efficient frontier that is supposed to guide us for asset allocation... But several problems remain:

1. Usual critics:
   (a) returns are not gaussian (depends on sampling frequency)
   (b) with exponential utility, constant risk aversion:

   $$RA = -\frac{U''(x)}{U'(x)} = a$$  \hspace{1cm} (6)$$

2. but the main problem is linked to the measurement of the parameters:
   (a) variances; one period framework: which model to measure
$\sigma_{t+1}|t$? In a multiperiod framework: how to predict volatility on that period?

(b) correlation; correlation strongly vary over time! For assets: positive. During crisis $\rightarrow$ goes up. How to account for this? How to predict it? Impact on the convexity of efficient frontier.

(c) expectation; what really makes money is the expected return forecast, because the earning of the strategy is worth:

$$\log \frac{P_T}{P_t} = \sum_{i=t+1}^{T} \log \frac{P_i}{P_{i-1}} = \sum_{i=t+1}^{T} r_i$$  \hspace{1cm} (7)

This last quantity: hard to compute/predict (at least with simple models). With ARMA-like: impossible (EMH). Notable exceptions:

(a) SETAR-like models...

(b) or MS models
But: computationally intensive and not stable though time...

For these reasons: the first attempt is simply to build hedging portfolio strategies.
1 Portfolio Insurance
Portfolio Insurance

Main idea: hedging strategy with equities, allowing to make the most of favorable evolutions of the markets.

A few words of vocabulary:

- Long position in a security means
  - the holder of the position owns the security
  - will profit if the price of the security goes up.
- Short position / shorting / short selling in a security means
  - the holder does not own the asset but sold it anyway
  - expect a drop of the security price.

From now on, two types of assets:

- a risky asset whose spot price at time $t$ is $P_t$
- a riskless asset whose price is $B_t = e^{-r_f(T-t)}$, with $T$ the maturity of the riskless asset.
Mainly, two types of strategies:

- **Buy stock as they rise and sell stock when they drop** are known as **convex strategies** (yield convex payoff)
- **Buy stock as they drop and sell stock when they rise** are known as **concave strategies** (yield concave payoff)

Yield different results, depending on market conditions:

- for oscillating markets, concave strategies are OK: strategies building on reversals
- for trendy markets, convex strategies are OK: make the most of directional evolutions

Convex strat. still require to have a **general idea** about the upcoming evolution of assets, BUT include insurance to hedge from big losses.
Strategies

Several types of basic convex strategies:

1. Buy & Hold: linear strategy hold until the fixed time horizon: no hedging (proceed with finger crossed).
2. Stop Loss strategy: switching from risky to riskless asset depending on a target threshold.
3. Constant Proportional Portfolio Insurance: constant proportional risk exposure that is a function of a cushion.
4. Option Based Portfolio Insurance: replicate the payoff of underlying + put to ensure a terminal value $K$. 
A concave strategy: **constant mix strategies**: keep a constant exposure to risky asset.

Each of these strategies:

– is based on a self-financing portfolio: no cash input on the fly
– have a fixed time horizon $T$
– are often represented as algorithm
1.1 Buy & Hold

– Not really a strategy.
– $\alpha$ number of shares, $N$ quantity in riskfree rate
– Portfolio value at time 0 :

$$Q_0 = \alpha P_0 + N$$  \hspace{1cm} (8)

– Final value of the portfolio :

$$Q_T = \alpha P_T + Ne^{r_f(T-1)}$$  \hspace{1cm} (9)

– Strategy makes the most of trends.
– Unaffected by volatility or correlation.
– Still : you have to know the future evolution of the risky stock
1.2 Stop Loss strategy

– Full investment in risky asset
– Define

\[ M_0 = xP_0 e^{-r_f T}, \]  
(10)

the minimum target required by the investor.
– \( x \) depends on risk appetite.
– Then : \( M_t \) evolves as

\[ M_t = xP_0 e^{-r_f (T-t)}, \]  
(11)

fixed Time varying

– Strategy :

1. Start with full investment in risky asset
2. If \( P_t < M_t \), the portfolio is invested in the riskless asset
3. If \( P_t > M_t \), the portfolio is invested in the risky asset
– With small daily variations, the minimum value of the portfolio is expected to be $M_0$.
– Unfortunalty, with jumps, the hedging constraint is often violated.
– Beware transaction costs / bid-ask spreads
cbind(x, strat, matrix(mini_ini, n, 1))
cbind(x, strat, matrix(mini_ini, n, 1))

0 100 200 300 400 500
80 90 100 110

1.3 Constant Mix Policy

- Concave strategies: make the most of price reversals. OK for High Volatility markets
- **Main idea**: keep constant risk exposure to the risky asset.

Define $S_t$ the absolute position in the risky asset at time $t$:

$$ RAP_t = N_t^P \times P_t $$  \hspace{2cm} (12) 

Define $V_t$ the value of the portfolio with factor loadings chosen at time $t$:

$$ V_t = RAP_t + RFP_t, $$  \hspace{2cm} (13) 

with $RFP_t$ the risk free asset position.
Purpose of the method:

Keep \( \frac{RAP_t}{V_t} = \alpha, \quad \alpha \in [0, 1] \)

\( \alpha \) is the risk exposure.

Algorithm:

1. Set \( V_0 \) such that the exposure is \( \alpha \)
2. Compute \( V_{t+1|t} \) the portfolio at time \( t + 1 \) given loadings chosen at time \( t \):

\[
V_{t+1|t} = RAP_t \times \frac{P_{t+1}}{P_t} + RFP_t \times e^{rf \times \Delta t}
\]  

(14)

Now \( \Rightarrow \) the constraint stands a good chance to be violated:

- need to change the loadings to ensure it again
- with a self financing portfolio
3. Rebalance positions such that:

\[ \text{RAP}_{t+1} = \alpha V_{t+1|t} \quad (15) \]

\[ \text{RFP}_{t+1} = V_{t+1|t} - \text{RAP}_{t+1} = (1 - \alpha) V_{t+1|t} \quad (16) \]

And go back to step 2.

A few comments:

– Unlike BH and SL → a concave strategy
– Define the number of stocks in portfolio:

\[ N_t = \frac{\text{RAP}_t}{P_t} \quad (17) \]

– Number of stock evolve contrary to the value of the stock since:

\[ \alpha = \frac{N_t P_t}{V_{t|t-1}} \quad (18) \]

\[ = \frac{N_t P_t}{N_{t-1} P_{t-1} + \text{RFP}_{t-1}} \quad (19) \]
⇒ the higher \( \frac{P_t}{P_{t-1}} \) and the lower \( N_t \)

**With constant mix strategies** :

- when the stock rise : sell because hoping on reversal
- when the stock drop : buy because hoping on reversal

⇒ Locally the expected payoff of the strategy is concave. Higher when a lot of volatility expected
cbind(x[2:n], port[2:n])

Index
Q[2:n]
cbind(x[2:n], port[2:n])

Index
Q[2:n]
1.4 CPPI

A more general approach to controlling risk exposure in the portfolio: the constant proportion portfolio insurance

Define:

– The floor $F$: minimum amount of money to be recovered in the end
– The cushion $C_t = V_t - F$
– The multiple $m$ that controls risk exposure
– Position in the risky asset: $RAP_t = m \times C_t$

$m$ controls the risk exposure of the portfolio across dates.
The strategy is then:

1. start with $C_0 = V_O - F$
2. then $RAP_0 = m \times C_0$ and $RFP_0 = V_0 - RAP_0$
3. Again,
   \[ V_{1,0} = RAP_0 \times \frac{P_1}{P_0} + RFP_0 \times e^{r_f \times \Delta t} \]  
4. The new cushion : $C_1 = V_{1,0} - F$
5. New position in the RA : $RAP_1 = m \times C_1$
6. And the new portfolio : $V_1 = RAP_1 + (V_{0,1} - RAP_1)$
7. Go back to step 3
Remarks:

- The CPPI is a convex strat. \( m > 1 \): when \( P_t \) goes up, position in the risky asset increases.
- When \( 0 < m < 1 \) and \( F = 0 \), constant mix strategy:

\[
m = \frac{RAP_t}{V_{t,t-1}}
\]  

(21)

- When \( m = 1 \), BH strat. with \( F \) in the riskless asset.

The strategy allows the investor to take part in local trends:
- Buy when prices rise
- Sell when prices fall
Asset/Port

Cushion

cbind(x[2:n], port[2:n], matrix(F, n − 1, 1))

RAP/Asset

RQ

RAP/V

alpha

Q

0.50 0.55 0.60

0.25 0.30 0.35 0.40 0.45 0.50

0 100 200 300 400 500

0 100 200 300 400 500

0 100 200 300 400 500

0 100 200 300 400 500

0 100 200 300 400 500

0 100 200 300 400 500

0 100 200 300 400 500

0 100 200 300 400 500

0 100 200 300 400 500
1.5 OBPI

From Leland and Rubinstein (1976). Main idea here:

- again, investing in risky and riskless asset
- to mimic a put payoff $\max(K - S_t, 0)$ plus the risky asset.

Basically:

\[
\text{Put + Underlying = Portfolio hedge}
\]

Easy to check:

\[
P_T + (K - P_T)^+ = \begin{cases} 
K & \text{when } P_T < K \\
P_T & \text{when } P_T > K 
\end{cases}
\]

(22)

Insurance to have at least $K$ in the end.
Question: portfolio loadings for today, with the previous final payoff?

Under no arbitrage arguments:

\[
e^{-r_f(T-t)} \mathbb{E}^Q \left[ P_T + (K - P_T)^+ | \mathcal{F}_t \right] = (P_t + Put(K, t, T, P_t))
\] (23)

Useless to buy a put, just have to replicate the BS formula:

\[
Put(K, P, t, T) = e^{-r_f(T-t)} \mathbb{E}^Q [(K - P)^+]
\] (24)

\[
= -N(d_1) P_t + KN(-d_2) e^{-r_f(T-t)}
\] (25)

The position:

\[
P_t + Put(K, P, t, T) = 1 - N(d_1) P_t + KN(-d_2) e^{-r_f(T-t)}
\] (26)
Problem: the portfolio has to be self-financed!

⇒ Using the loadings as weights instead of quantities.

Algorithm:
- Start with \( V_0 = (1 + N(d_1))S_0 + Ke^{-rfT}N(-d2) \) Then for any date:
- as usually: \( V_{t+1,t} = RAP_t \times \frac{P_{t+1}}{P_t} + RFP_t \times e^{rf\Delta t} \)
- "Optimal" portfolio:

\[
V^*_{t+1} = (1 - N(d_1^*))P_{t+1} + KN(-d_2^*)e^{-rf(T-t-1)}
\] (27)
– Exposure to risky asset $\alpha$ comes from:

$$V_{t+1} = V_{t+1,t} \times \frac{V^*_t}{V^*_t}$$  \hspace{1cm} (28)

$$= V_{t+1,t} \left( \frac{(1 - N(d_1^*))P_{t+1}}{V^*_t} \right) + \frac{KN(-d_2^*)e^{-rf(T-t-1)}}{1-\alpha} \frac{V^*_t}{V^*_t}$$  \hspace{1cm} (29)

– Thus, $RAP_{t+1} = \alpha V_{t+1,t}$ and $RFP_{t+1} = (1 - \alpha)V_{t+1,t}$
– The new portfolio: $V_{t+1} = RAP_{t+1} + RFP_{t+1}$
Loadings have to be computed and the port. rebalanced to ensure the option-like hedge.

Convex strategy

Important: the hedge holds on average!
cbind(x, port, matrix(K, n, 1))

Port/P

alpha

Quantity of RA

position_action/x
2  Factor Based Method Analysis
Purposes of the Section

This section is devoted to dimension reduction in financial markets:

– Globally: a lot of information to be dealt with that may be redundant
– Arbitrage tracking mostly based on factor based model of the financial markets (CAPM, APT...)

⇒ But these factors are unobservable or observed in a noisy way

Need for methods to estimate these factors in order to:

– control for risk for large portfolios
– find minimum variance portfolios
– improve the understanding of the fine financial market dynamics
– ...
Organization of this subsection:

1. Crash introduction to Principal Component Analysis
2. Use for minimum variance portfolio building
3. Use for term structure analysis in swap rates
2.1 A Crash Introduction to PCA

Principal Component Analysis = a way around dimension reduction problems

Basics on PCA

Let $X_t = (x_{1,t}, x_{2,t}, \ldots, x_{n,t})^\top$, $t = 1, \ldots, T$ be the matrix of observations to be modeled

Remarks:

1. $X_t$ is covariance stationary: the covariance process does not depend upon $t$, but only on the sampling frequency:

$$\Sigma_t = \Sigma_{t+1} = \ldots$$ \hspace{1cm} (30)

- E.g. GARCH-like models are not covariance stationary, just like financial time series (in general)
- GARCH models are conditionally covariance stationary ⇒
filtered series are stationary

2. $X_t$ can be such that $T << n$ and PCA still works

3. PCA are anyway applied to financial time series with important consequences on asset allocation.

4. Most of the time, the time series are considered to be (time) locally stationary.

5. In Meucci’s book: ”time invariance” for stationarity

**Assume that:**

1. $X_t$ has zero mean

2. $X_t$ has unit variance
Correlation matrix :

\[ C = T^{-1}X^\top X \]  \hspace{1cm} (31)

PCA is based on eigenvalue/vector decomposition of \( C \)

**Purposes of PCA**

\[ X_t \Rightarrow F_t = (F_{1,t}, ..., F_{k,t}) , \text{ such that } k << n \]  \hspace{1cm} (32)

**Requirements of PCA**

- Form factors \( F_i \) named principal component
- That best summarize (possibly redundant) information in the dataset
- Quantity of information = variability of the dataset
- Factors must be ordered : first factor = explain most of the information
- Principal component are uncorrelated with each other

Used in many fields (chemometrics, econometrics, finance, biology...
The spectral theorem in short

Let $S$ be a symmetric positive definite matrix, i.e.:

1. $S = S^\top$  
   \hspace{1cm} (33)
2. $|S| > 0$ or $\exists v \in \mathbb{R}^n : v^\top S v > 0$  
   \hspace{1cm} (34)

then $S$ can be decomposed as:

$$S = D\Lambda D^\top,$$  
\hspace{1cm} (35)

with

$\Lambda = diag(\lambda_1, \ldots, \lambda_n)$ with $\lambda_1 > \ldots > \lambda_n$\hspace{1cm} (36)

$D^\top = D^{-1} \iff DD^\top = I_n$\hspace{1cm} (37)

$Tr(S) = Tr(D\Lambda D^\top) = Tr(\Lambda) = n$\hspace{1cm} (38)

$D$ is a rotation matrix\hspace{1cm} (39)
Solution to the PCA problem

– $C$ fulfill the spectral theorem requirements, thus:

$$ C = D \Lambda D^\top $$ (40)

– The $i^{th}$ optimal factor is given by:

$$ F_i = X D_i, $$ (41)

with $D_i$ the $i^{th}$ column of $D$.

– $V(F_i) = T^{-1} F_i^\top F_i = D_i^\top X^\top X D_i = D_i^\top C D_i = \lambda_i$

– Similarly : $\rho(F) = \frac{F^\top F}{T} = \Lambda$, which is diagonal $\Rightarrow$ no correlation between factors

Remark

Since $D$ is a rotation matrix :

$$ \Rightarrow \text{Factors are rotations of the original dataset} $$
Applications

Factor analysis is usually applied to:
- Interest Rates Swap (IRS) to recover the usual three factors of the yield curves
- Implied volatilities across moneynesses to recover the three factors in implied volatility dynamics
- Equity stocks to build riskless portfolios
- ...

Most of the time:
- better understanding of financial asset dynamics
- better understanding of risk level implicit in positions
- help build strategies
- help build hedge against specific movements
- help build funds with a controlled risk exposure
- ...
2.2 PCA applied to swap rates

\[ r_t(\tau) = \text{swap rate with time to maturity } \tau \text{ on date } t \]

**Comments**

- Swap rates used to build term structure of interest rates
- Higher risk than for bills (no collateral)
- But consistent with interbank loans and CB target rate
- No ZC yield, but coupon paying par yield

**Important point:**

1. Swap rates are not second order stationary
2. Daily variations can be assumed to be second order stationary in a first approach
3. But they are not! Second order moments and comoments vary through time...
First case: the EURO curve

- New market
- German dominance
- Progressive learning of the market
- Different periods of vol. due to diff. central bankers + market conditions
- ...

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Term structure of interest rates
Second case: the US case

- Old market
- No particular dominance
- from 2000 to 2007: one change in CB
- ...
Term structure of interest rates

1:length(dates)

x


1:length(dates)
Cum. first three fact.

1Y–5Y–10Y swap rates
Lessons from the US and Euro rates

- Three factors in the yield curve:
  1. Level (85%)
  2. Slope (10%)
  3. Convexity (5%)

- Factors are not stable:
  - importance varies through time
  - composition seems to switch depending on dates

  Why? mainly: time varying correlation that defines the eigenvectors and eigenvalues

- Important for risk management of bond portfolio: main blow comes from the level

- Importance of strategic allocation: butterfly strategies yield the less volatile payoff. Beware of level position → will go through most of the market moves.
Steps further

Possible to summarize much more information:

– Global risk factor and the contribution of the US
– Enhance the global risk perception of the bank
– Allow for fine understanding of these factors
– Help build asset pricing models close to reality:
  • Vasicek, Cox Ingersoll Ross = one factor models (only level)
  • other affine models (Piazzesi (2001)) allow for more factors
Remarks

Similar results obtained for Implied Volatilities:

1. Alexander: departure from at the money implied volatility with a fixed time to maturity:
   - With volatilities: need to deal with additional difficulties:
     • No fixed time to maturity products avail.
     • The underlying varies a lot: distance from strike to spot unstable. ⇒ cannot compare raw implied volatilities through time
     • Alexander compares departure from ATM vol: series closer to stationarity
   - Results: again three factors: parallel shifts, asymmetry and curvature of the smile.
2. Da Fonseca and Cont:
   – another approach taking explicitly the dimensional problem into account:
     
     \[
     \text{Time to expiry} \times \text{Calendar time} + \text{Fixed TTM} + \text{Fixed moneyness}
     \]
   
   – Use Karhunen-Loève decomposition to take this dimensionality into account.
   
   – Again, three factors: level of the smile term structure, potential asymmetry and curvature.
Final remarks on stationarity problems

Several way around the problem :

1. Modelling the eigenvalues may not be the right answer :
   – need to maintain positivity
   – tricky/non stationnary dynamics of the eigenvalues
   – no a priori model

2. Modelling the correlations = a better way, since the quest for
definite positive dynamic correlation matrices is achieved both a
continuous and discrete time :
   – Discrete : Dynamics Conditional Correlation Model of Engle
     (2002)
     • GARCH dynamic for volatility
     • Independent model for correlation in a autoregressive way
     • Extended in Capiello, Engle and Sheppard (2003) to accomodate
       leverage and asymmetric correlation effects.
- QML estimation is easy to implement (estimation yet tricky)
- makes it possible to forecast covolatility matrices
- **Wishart Affine Stochastic Correlation Model (WASC)** of da Fonseca, Grassello and Tebaldi (2006) in a continuous time setting:
  - Multidim. Heston-like model
  - Ok for asymmetric correlation effect and leverage
  - Can be used for optimal portfolio choice, term structure modelling...
  - Closed form expression for the dynamics of the first eigenvalue of the correlation matrix
  - ...

Yet = attempts to model the eigenvalues proved to obtain interesting results.
### 2.3 PCA for portfolio management

**Remark**

In PCA analysis, the eigenvalues measure the level of risk associated to the linear combination of the assets in the portfolio:

\[ F_i = R \times D_i, \]

with \( R \) the return matrix, \( D_i \) the \( i^{th} \) eigenvector.

\[ \sigma(F_i) = \lambda_i. \]  \hspace{1cm} (43)

- \( D_i \) = portfolio loadings
- \( F_i \) = portfolio value
Several Trading Strategies

– For large dataset: final eigenvalues are closed to zero $\Rightarrow \sim$ riskless portfolio.
  
  In a market with no arbitrage, expected return of such portfolio should be riskfree rate
  
  Borrow risk free rate + Long position in $F_i$

– Some portfolios have low risk level but are long short positions $\Rightarrow$ low fund level required (asset management)? Interesting because:
  
  1. factors are orthogonal: port. is hedged against the remaining risk factors
  2. even with positive correlation: diversification effect
  3. Long/short: limited amount on initial investment (but possible leverage)
Backtested returns for PCA portfolio

sigma
abs(mu)
Main results

- Port. 1: global port.
- Port. 2: SP500
- Port. 3: FTSE/DAX vs. CAC
- Port. 4: CAC vs. DAX (long/short portfolio)

Hard to find a group of port. that clearly beat the market but:
- No idea about the future direction: hard to predict the sign to associate to $D_i$
- No risk free rate used with PCA
- Still possible to combine it with port. insurance technics
- ...
3 Pairs trading and cointegration

General idea

In the previous section : DAX/CAC long short port.
⇒ long in one asset and short in the other for minimum capital requirement

General purpose of this method :

Bet reversals in long term relationships

Ex : when you know the usual spread between DAX and CAC is $x$ and now it is worth $y > x$, you can bet on the reversal to the long term value

Problems

– how to estimate this long term value ?
– is this long term value stable ?
1. To the first question: cointegration.

**What is cointegration?**
- If $X_1$ and $X_2$ are I(1) time series (first difference are stat.time series)
- Estimate with OLS $X_1 = \alpha + \beta X_2 + \epsilon$
- If the estimated $\epsilon_t$ is I(0) then we say $X_1$ and $X_2$ are cointegrated

The estimated $\epsilon$ gives us clues around:
- how far are indexes from their long term relationship?
  **answer**: far if $|\epsilon|$ is big
- what is the speed of mean reversion to it?
  **answer**: estimate AR(P) process on $\epsilon$. For AR(1):

$$\epsilon_t = \theta \epsilon_{t-1} + \nu_t,$$

(44)

if $\theta$ is small: fast
if $\theta$ is close to 1: slow

Trading idea

When the residual is big enough, trade on the collapse of the indexes to their long term relation, if the mean reversion speed is high enough.

Problems

- How to choose the estimation window?
- How stable are long term relationship?
- How to choose the investment horizon?
- ...
A simple example on DAX and CAC again:

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Std. Error</th>
<th>t value</th>
<th>P-val</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>-3.608e+02</td>
<td>1.388e+01</td>
<td>-26.00</td>
<td>&lt;2e-16</td>
</tr>
<tr>
<td>CAC</td>
<td>1.231e+00</td>
<td>3.807e-03</td>
<td>323.46</td>
<td>&lt;2e-16</td>
</tr>
</tbody>
</table>

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 369.9 on 4563 degrees of freedom
Multiple R-Squared: 0.9582, Adjusted R-squared: 0.9582
F-statistic: 1.046e+05 on 1 and 4563 DF, p-value: < 2.2e-16
Spread to the long term relation

eps

janv. mars

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