

Portfolio Management in the CAPM framework

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The purpose of this short class is to introduce key concepts on portfolio management, fund management and alpha generation. The theoretical background of the class is the Capital Asset Pricing Model. Let P_t^i be the price of the i th stock at time t . Then, its return r_t^i is

$$r_t^i = \log \frac{P_t^i}{P_{t-1}^i} \quad (1)$$

also known as logarithmic return. Let P_t^M be the price of the market portfolio at time t . Then again its return is

$$r_t^M = \log \frac{P_t^M}{P_{t-1}^M}. \quad (2)$$

The risk free rate here is the return on a government bond written in the same currency as the stocks and the market portfolio. The yield on a daily basis is denoted r_t^f .

The CAPM relation is the following one

$$r_t^i = r_t^f + \beta^i (r_t^M - r_t^f), \quad (3)$$

with $\beta^i = \frac{\text{Cov}(r_t^i, r_t^M)}{V(r_t^M)}$.

From a statistical point of view, we assume the relation is not perfect and that the errors can be modeled through a distribution with expectation 0 and standard deviation σ^i . The relation now becomes:

$$r_t^i = r_t^f + \beta^i (r_t^M - r_t^f) + \epsilon_t^i, \quad \epsilon_t^i \sim N(0, \sigma^i). \quad (4)$$

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The assumed Gaussianity of the error process is only here to ease the derivation of the finite sample properties of the Ordinary Least Square estimator of the β^i parameter. This estimator is such that

$$SSE(\beta^i) = \sum_{t=1}^T (\epsilon_t^i)^2 \quad (5)$$

is minimum. The estimator that matches this condition is

$$\hat{\beta}^i = \frac{\sum_{t=1}^T (r_t^i - r_t^f)(r_t^M - r_t^f)}{\sum_{t=1}^T (r_t^M - r_t^f)^2}. \quad (6)$$

This estimator is only consistent in the case where there is no constant term in the regression. In such a case, this estimator is the exact formula given by the CAPM. When such a constant term exists, it captures an abnormal return of the stock i with respect to the market. The relation now becomes

$$r_t^i = \alpha^i + r_t^f + \beta^i(r_t^M - r_t^f) + \epsilon_t^i, \quad \epsilon_t^i \sim N(0, \sigma^i). \quad (7)$$

A matrix-based resolution is necessary. Let

$$Y^i = \begin{pmatrix} r_1^i - r_1^f \\ r_2^i - r_2^f \\ \vdots \\ r_T^i - r_T^f \end{pmatrix} \quad (8)$$

$$X^i = \begin{pmatrix} r_1^M - r_1^f \\ r_2^M - r_2^f \\ \vdots \\ r_T^M - r_T^f \end{pmatrix} \quad (9)$$

$$\theta^i = \begin{pmatrix} \alpha^i \\ \beta^i \end{pmatrix}. \quad (10)$$

The estimator is now

$$\hat{\theta}^i = ({}^tX^i X^i)^{-1}({}^tX^i Y^i) \quad (11)$$

that is very different from what is obtained in the "without constant term" case. To see this, you just need to break down the matrices. The strong interest in this statistical framework is that it is possible to test whether α^i is really different from 0, since the distribution of the estimates is known:

$$\hat{\theta}^i \sim N(\theta^i, ({}^tX^i X^i)^{-1}(\sigma^i)^2). \quad (12)$$

Now the question is: "how can I build a strategy that makes the most of the alpha generation?". The answer is simple: you just need to build a portfolio that is as less correlated as possible to the index returns. Say you pick stock i . Such a portfolio should be :

$$P = \theta^i P^i + \phi P^M, \quad (13)$$

so that

$$\partial_{r^M} (\theta^i r^i + \phi r^M) = 0. \quad (14)$$

Then ϕ must be chosen so that

$$\phi = \theta^i \beta^i. \quad (15)$$

Questions for the class

The class is based on the Eurostoxx 50 and the 3M German govt bond. We focus on the case where it is only possible to be long in the stocks and the index.

1. The fund management approach.

The first part of the questions mainly deals with the fund management approach. See the index as a benchmark fund and try to beat it by going over/underweight in its component.

- (a) Compute the returns of the stocks and of the index.
- (b) Fill in the blanks in the VBA function that computes the β for each stock.
- (c) Compute the β of each stock over the whole period. Compare the value and the sector to which these stocks belong and discuss which sector can be seen as defensive and offensive.
- (d) Compute these beta year by year. Comment the obtained results.
- (e) Suppose we are in 2005. Look at the weights of the market portfolio. From your previous analysis design a strategy to beat the benchmark index return over 2005. Which stock would you go under/overweight?
- (f) Do the same in 2008. Comment.

2. The second part focuses on a total return approach. Here, you wish to build market neutral portfolio that generates alpha.

- (a) Fill in the blanks in the alpha computation VBA function.
- (b) Compute the alpha, along with the statistical test for its significativity for each stock over the whole period, and compare it to a year by year approach.
- (c) Build in 2005 a portfolio that is neutral to the market that makes the most of your alpha analysis. Compare it to the risk free rate and to a buy and hold strategy of the index.
- (d) Build in 2008 a portfolio that is neutral to the market that makes the most of your alpha analysis. Compare it to the risk free rate and to a buy and hold strategy of the index.